

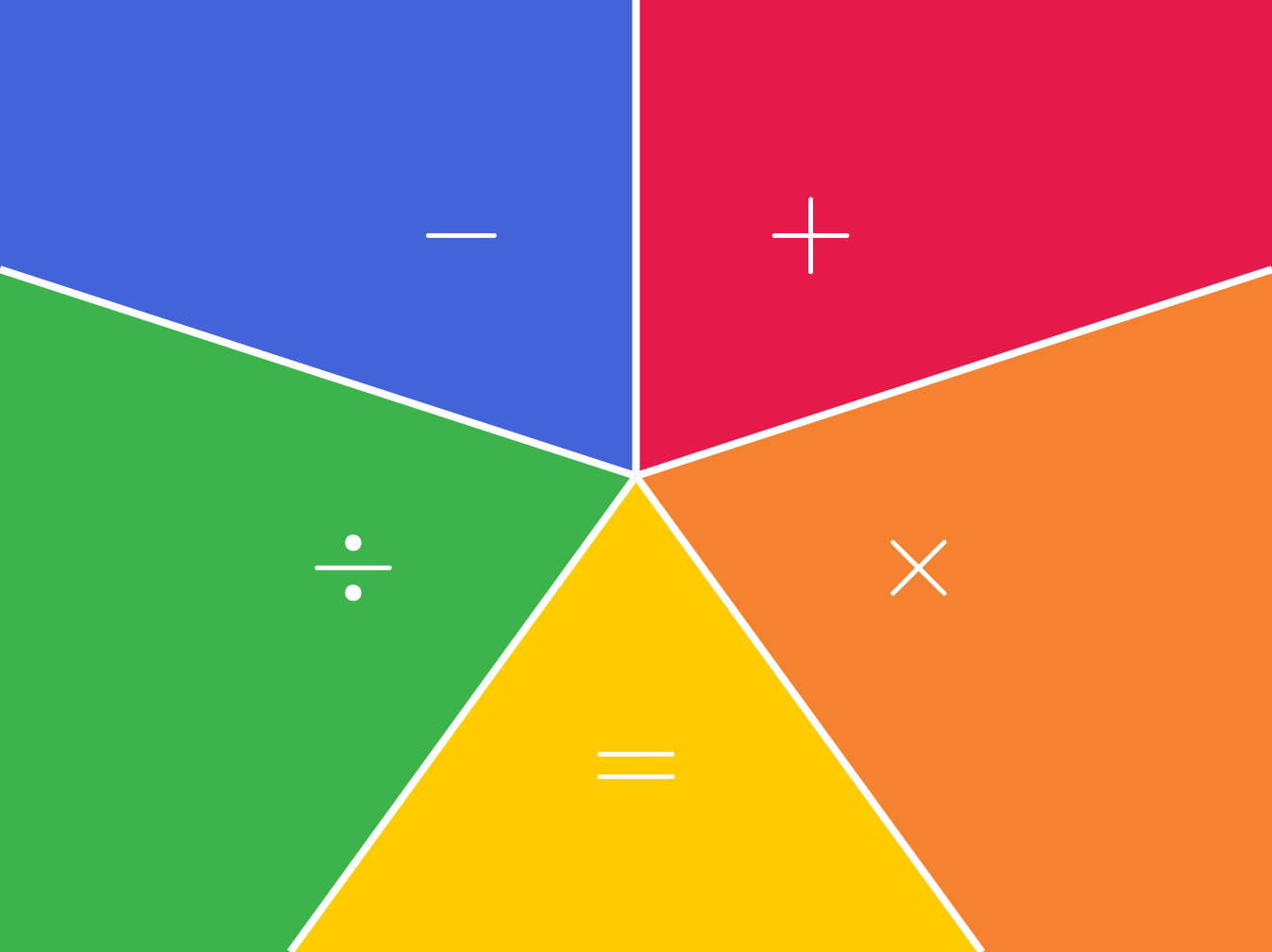
Combinatoria

Charles

Training Camp

Medellín 2026

Qué es? Contar cosas.



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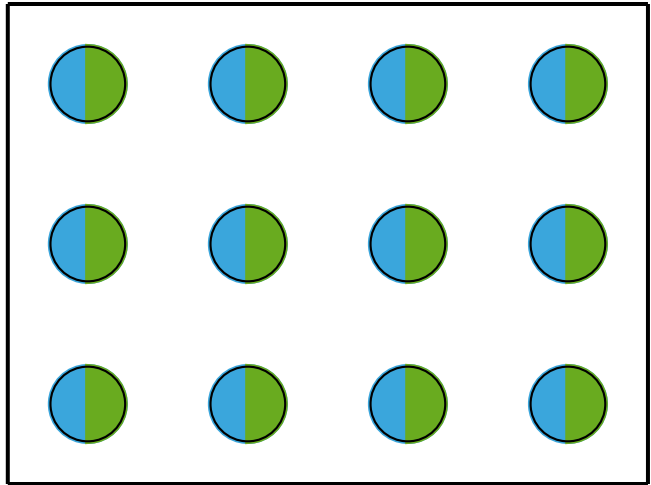
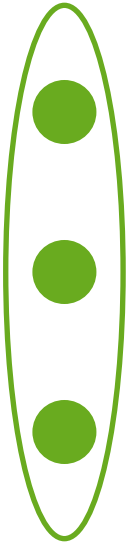
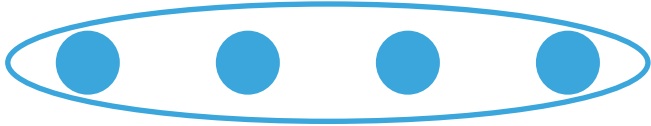
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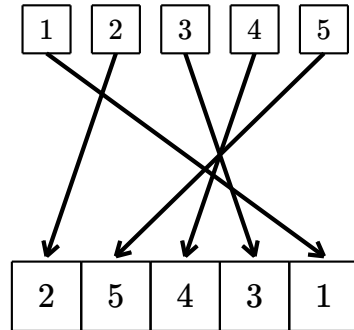
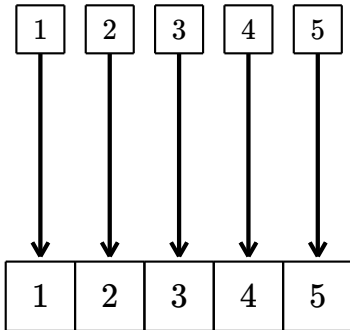
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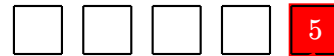
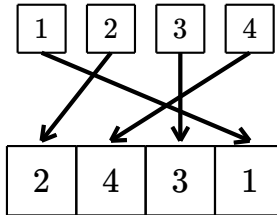
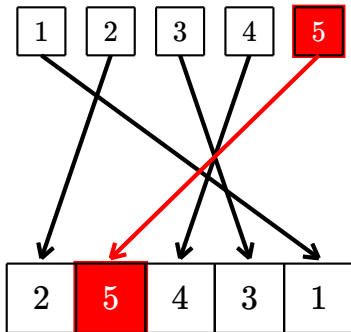
Producto Cartesiano



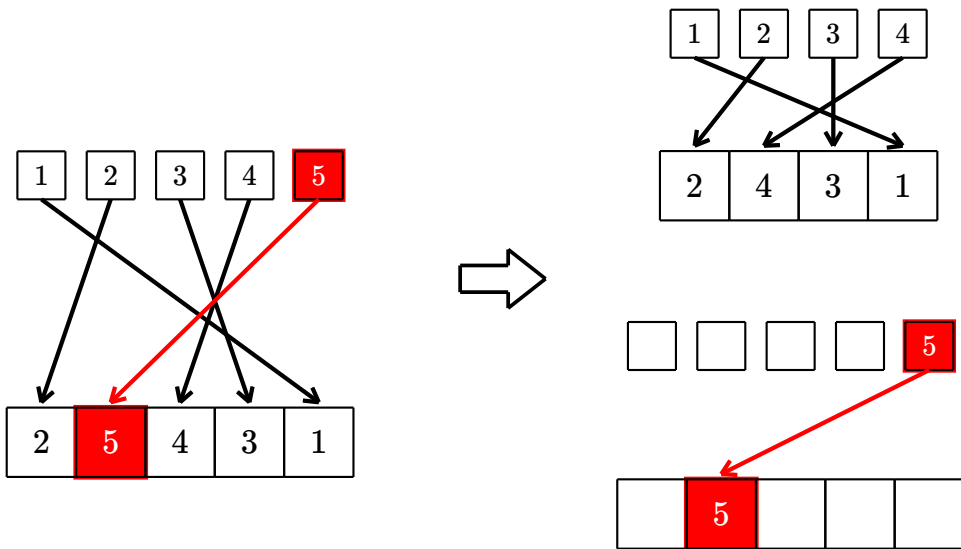
Permutaciones



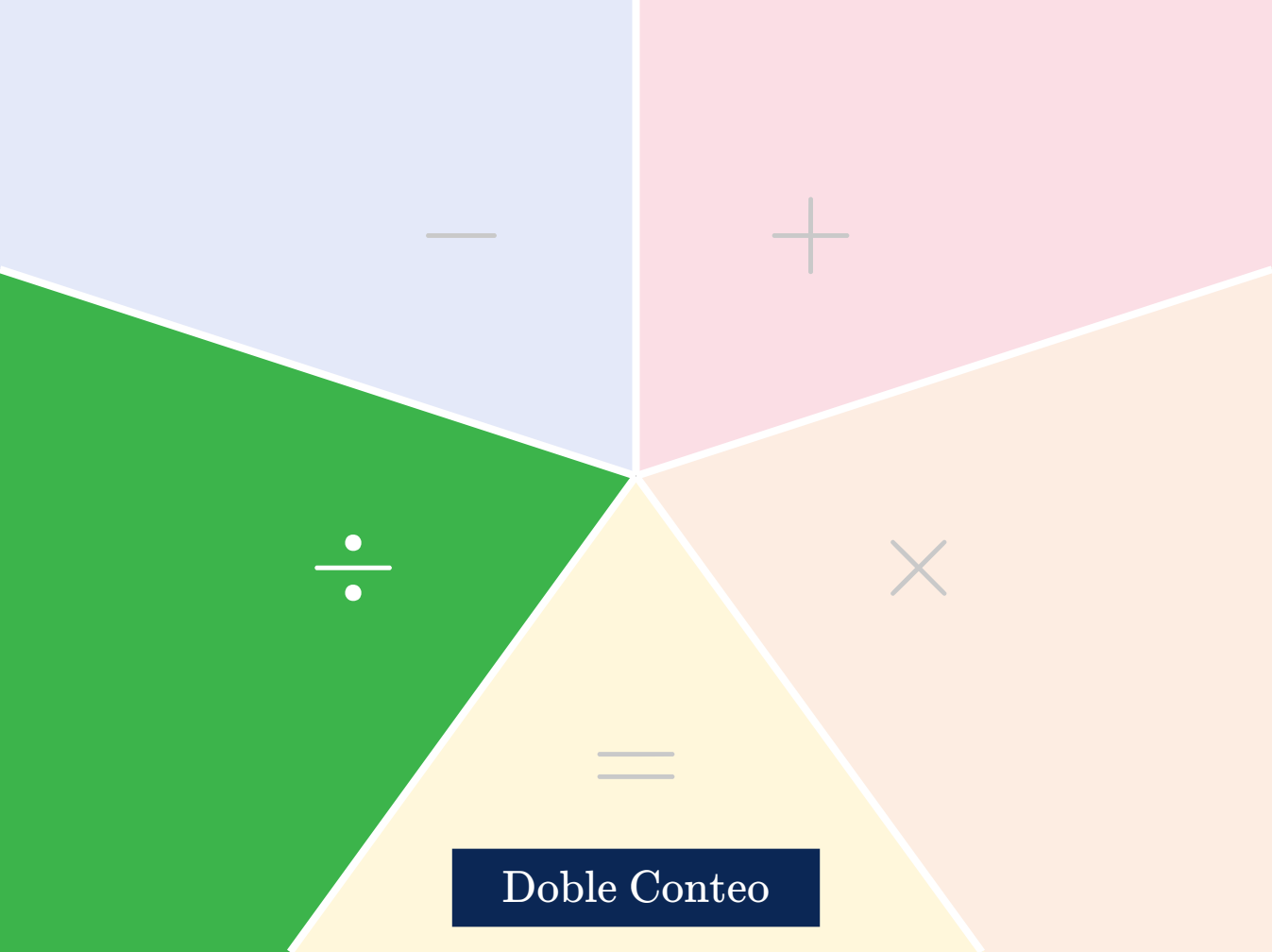
Conteo Inductivo



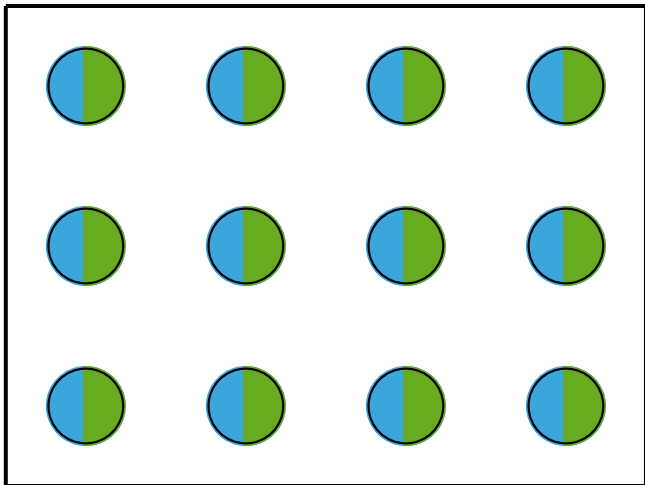
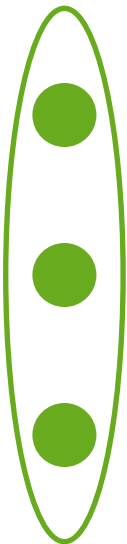
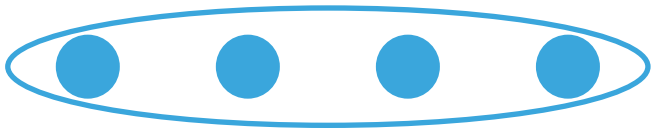
Conteo Inductivo



$$\text{perm}(5) = 5 \cdot \text{perm}(4)$$



Doble Conteo



Coeficientes binomiales

 x3

 x2

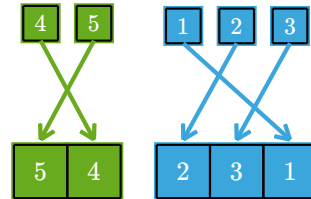
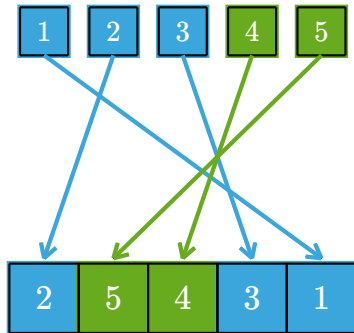
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Coefficientes binomiales

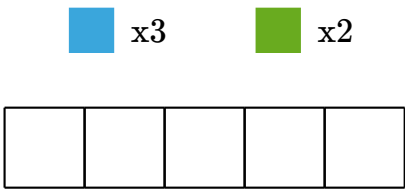
■ x3 ■ x2



Double Counting



Double Counting

$$\# \left(\begin{array}{cc} \color{blue}{\square} \times 3 & \color{green}{\square} \times 2 \\ \square & \square & \square & \square & \square \end{array} \right) \cdot 3! \cdot 2! = 5!$$
The diagram shows a horizontal row of five empty square boxes. Above the first three boxes is a blue square followed by the text 'x3'. Above the last two boxes is a green square followed by the text 'x2'. The entire diagram is enclosed in large parentheses, with a hash symbol to the left and the equation '.3! .2! = 5!' to the right.

$$\binom{5}{3} = \frac{5!}{3! \cdot 2!}$$

$$\binom{n}{k} = \frac{n!}{k! \cdot (n - k)!}$$

Kyoya and Colored Balls

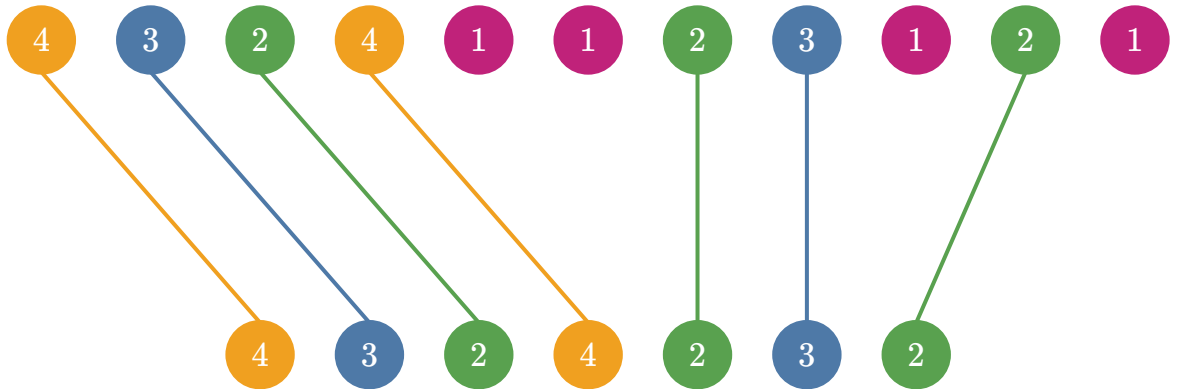
<https://codeforces.com/problemset/problem/553/A>

Se tienen k colores y c_i (con $1 \leq i \leq k$) bolitas de cada color. Se las quiere poner en fila tal que la última bolita del color i esté antes que la última bolita del color $i - 1$.



Kyoya and Colored Balls

Si sacamos los 1, queda un array válido con un color menos.



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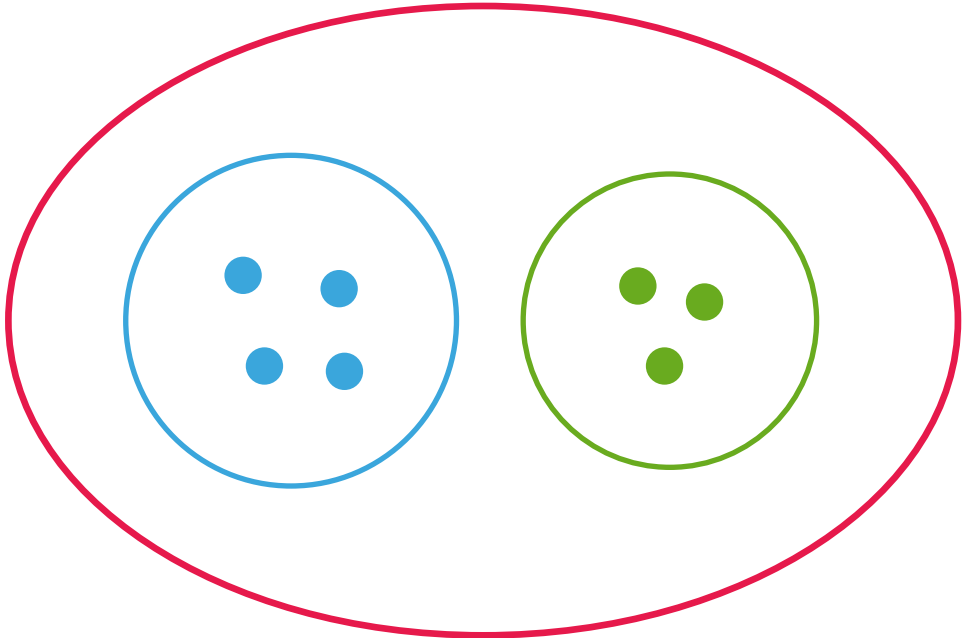
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Separación en Casos



Triángulo de Pascal

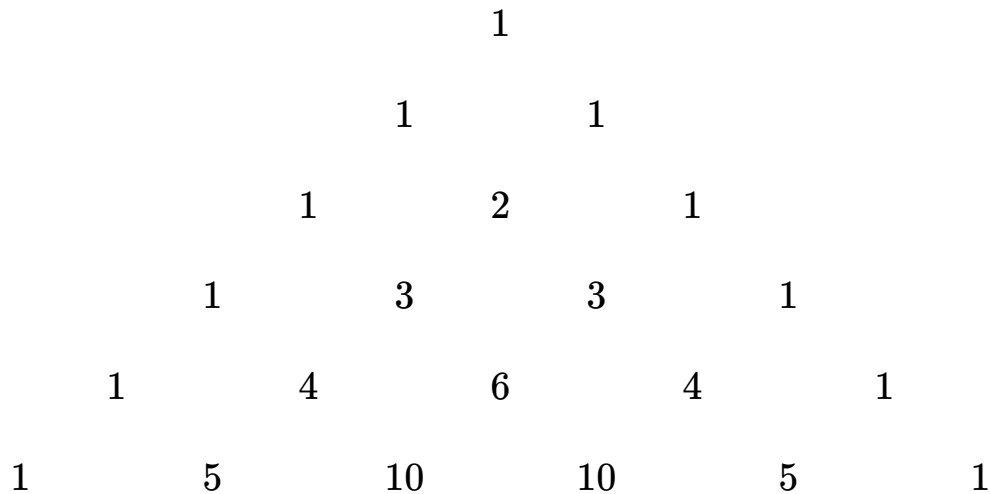


$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Triángulo de Pascal

$$\begin{array}{cccccc} & & & \binom{0}{0} & & & \\ & & & \binom{1}{0} & & \binom{1}{1} & \\ & & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} \\ & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \\ & \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} \\ \binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5} \end{array}$$

Triángulo de Pascal



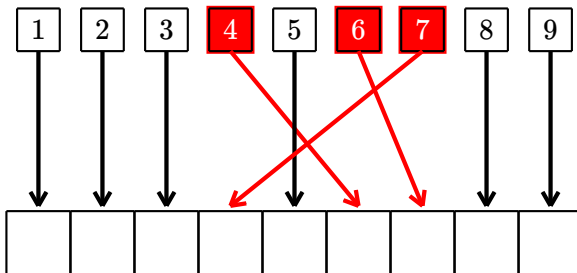
The image displays Pascal's Triangle, a triangular arrangement of numbers. Each row contains one more number than the row above it, starting with a single '1' at the top. The numbers in each row are the sum of the two numbers directly above them. The triangle is centered on the page.

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1	1	4		6		4	1	
1	5	10		10		5	1	

Problema de Ejemplo

Una permutación es casi identidad si todos excepto a lo sumo k de los números van a su propia caja.

$$n \leq 10^4, k \leq 4$$



$$\binom{n}{0} \cdot 0! + \binom{n}{1} \cdot 1! + \binom{n}{2} \cdot 2! + \binom{n}{3} \cdot 3! + \binom{n}{4} \cdot 4!$$

No Anda!



$$\binom{n}{0} \cdot D_0 + \binom{n}{1} \cdot D_1 + \binom{n}{2} \cdot D_2 + \binom{n}{3} \cdot D_3 + \binom{n}{4} \cdot D_4$$

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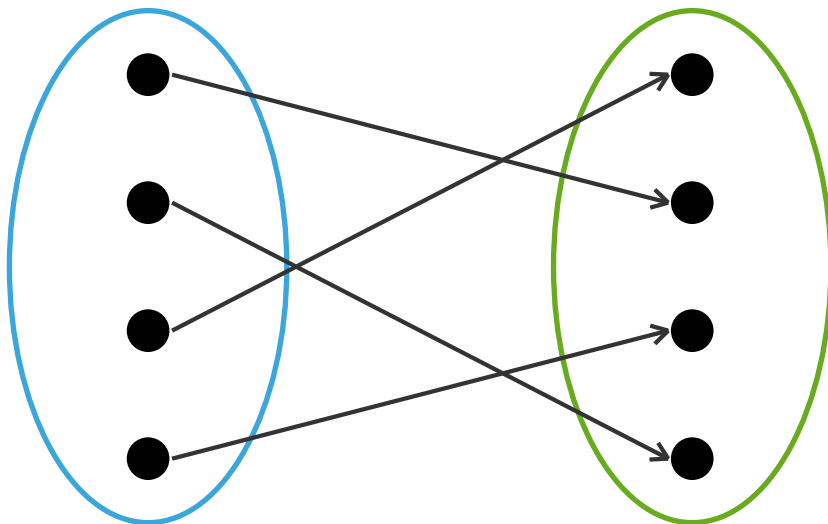
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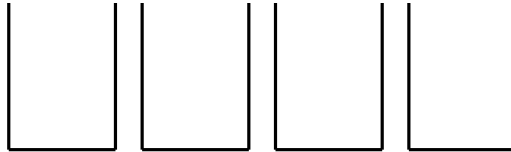
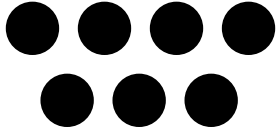
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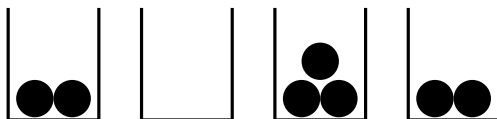
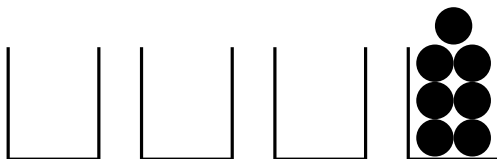
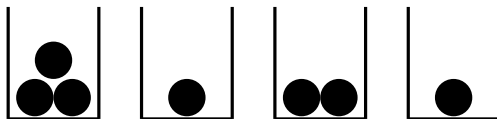
Isomorfismo



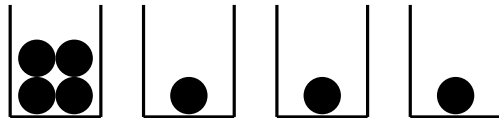
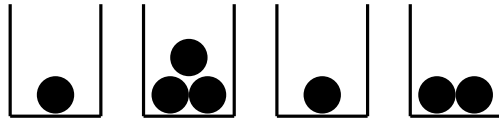
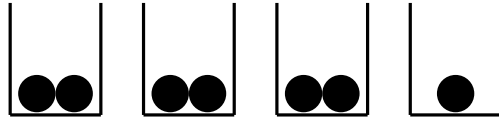
Bolitas y Cajitas

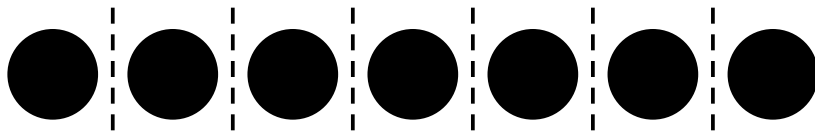


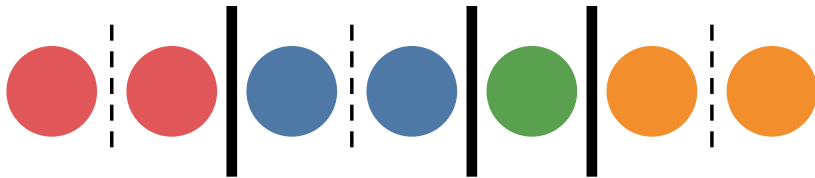
Bolitas y Cajitas

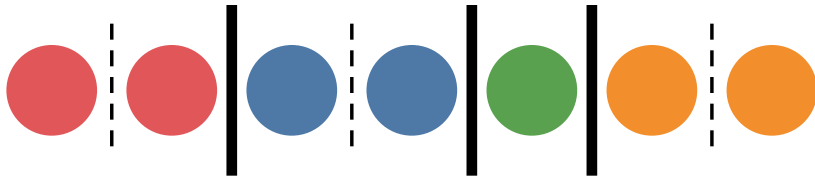


Bolitas y Cajitas (no vacías)



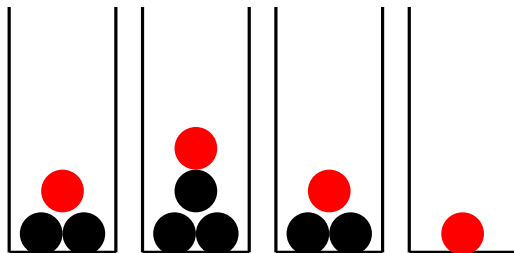






$$\text{ByCNV}(7, 4) = \binom{6}{3}$$
$$\text{ByCNV}(b, c) = \binom{b-1}{c-1}$$

Bolitas y Cajitas (con vacías)

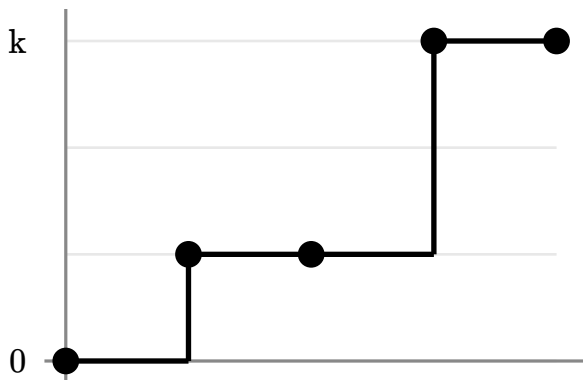


$$\text{ByC}(7, 4) = \text{ByCNV}(11, 4) = \binom{10}{3}$$

$$\text{ByC}(b, c) = \text{ByCNV}(b + c, c) = \binom{b + c - 1}{c - 1}$$

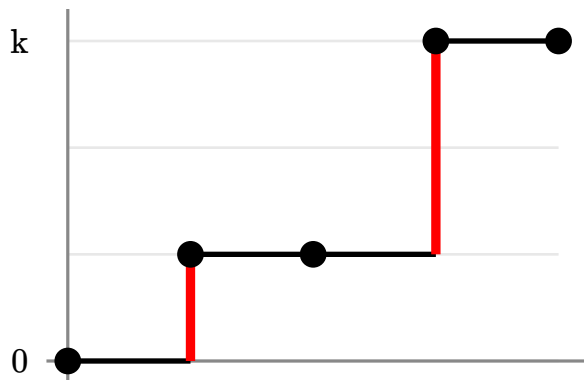
Secuencias no decrecientes

Contar las secuencias no decrecientes de largo n que empiezan en 0 y terminan en k .



Secuencias no decrecientes

La secuencia queda determinada por sus $n - 1$ saltos $d_i = a_{i+1} - a_i$:
son $n - 1$ enteros $d_i \geq 0$ que suman k .



$$\# = \text{ByC}(k, n - 1) = \binom{n + k - 2}{k}$$

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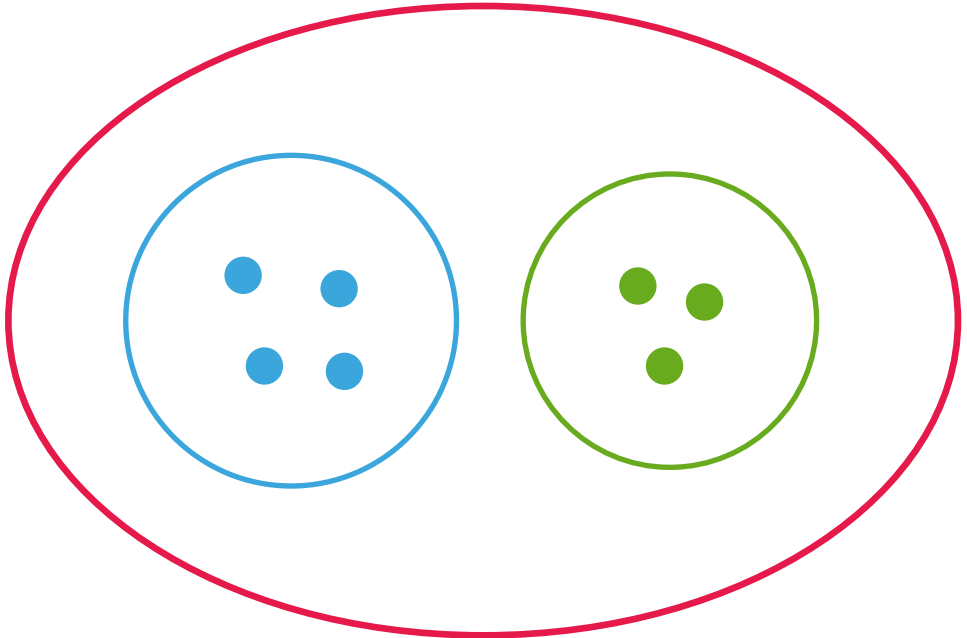
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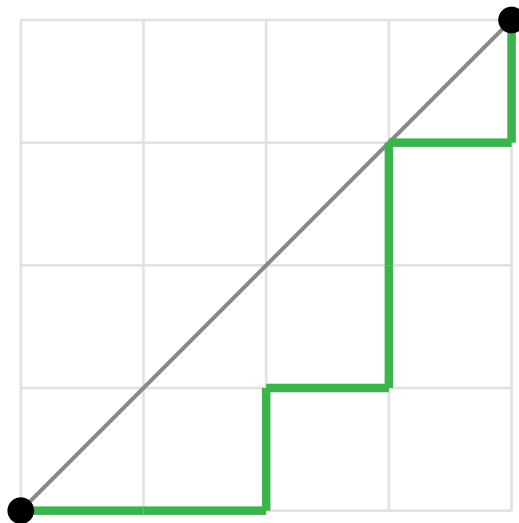
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Contar el Complemento



Contar el Complemento: Caminos de Dyck

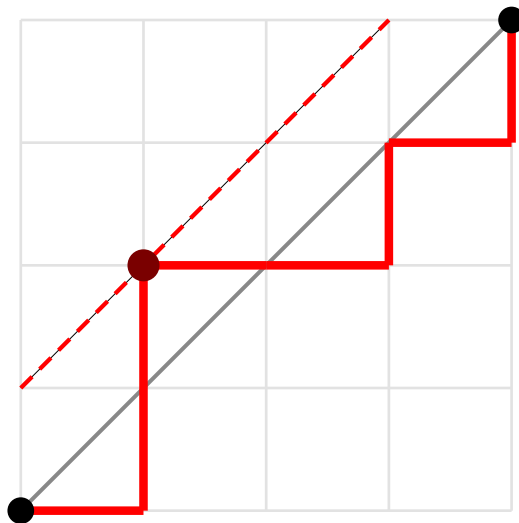
Un camino monótono de $(0,0)$ a (n,n) da pasos a la derecha y arriba.
Cuántos quedan siempre (débilmente) por debajo de la diagonal?



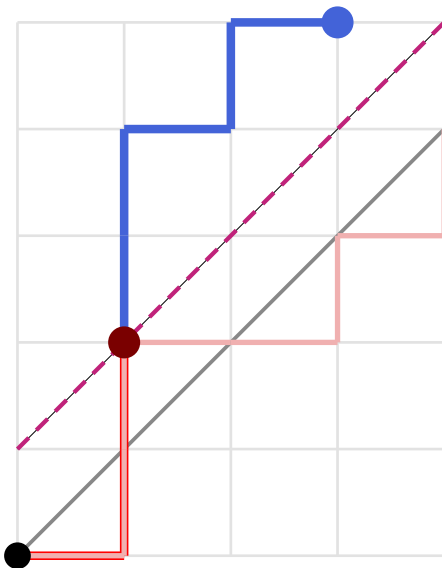
Contar el Complemento: Caminos de Dyck

En total hay $\binom{2n}{n}$ caminos monótonos.

Contamos los malos: los que cruzan, o sea tocan la recta $y = x + 1$.



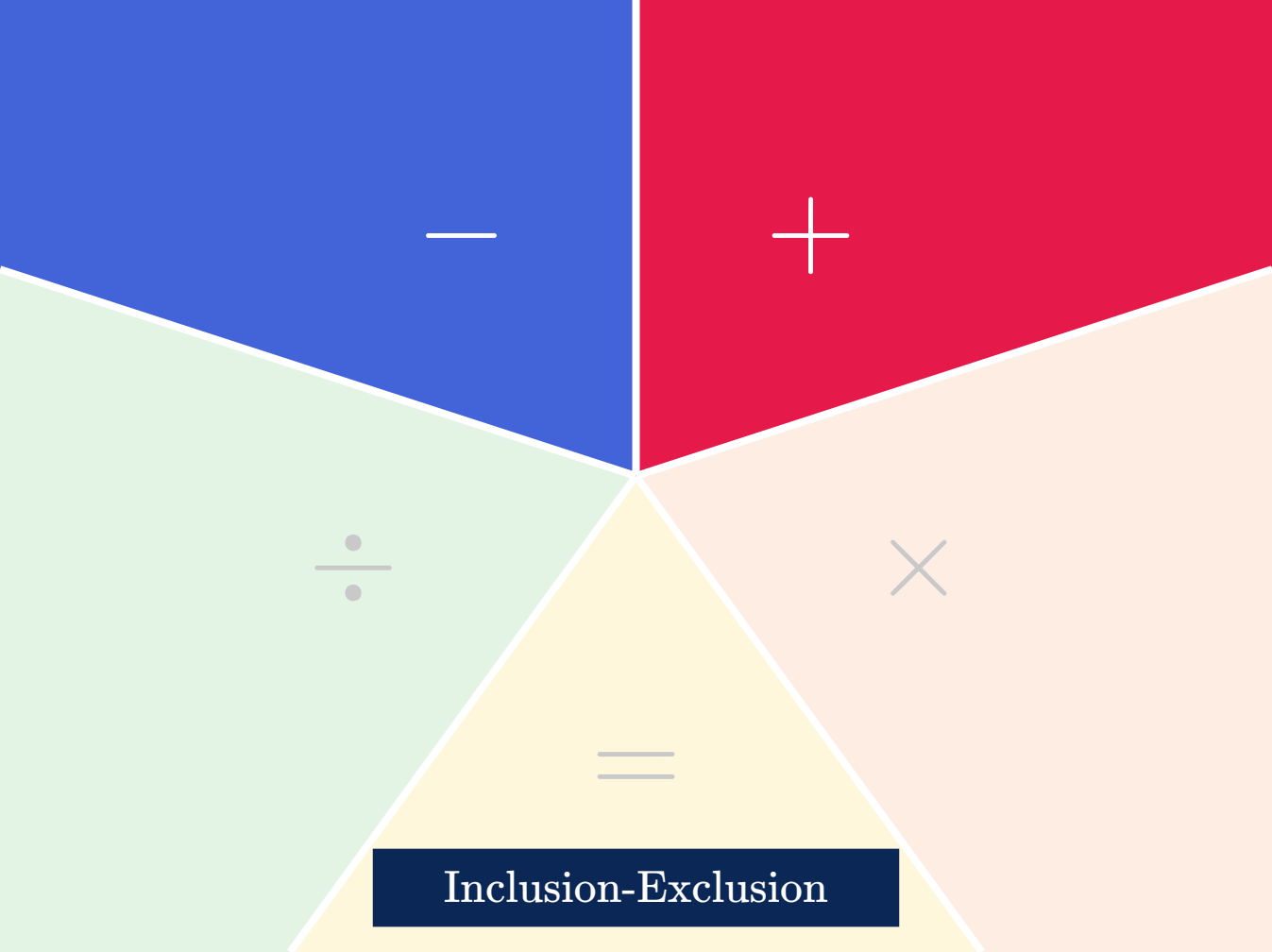
Contar el Complemento: Caminos de Dyck



$$\# \text{ buenos} = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1} \binom{2n}{n}$$

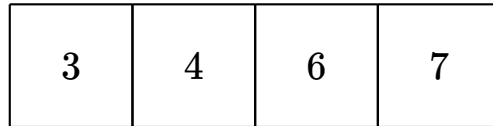
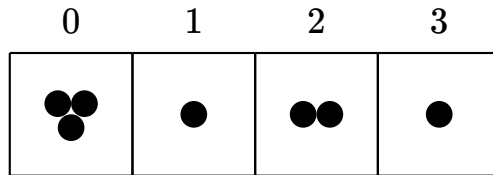
Break

10 min

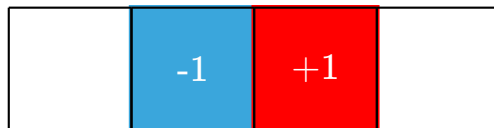
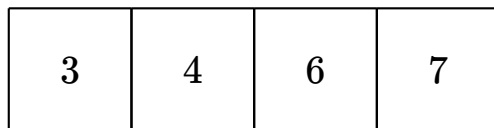
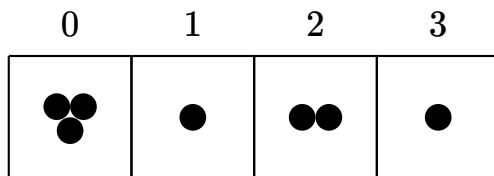


Inclusion-Exclusion

Inclusión Exclusión



Inclusión Exclusión



















$$A[i] = S[i] - S[i - 1]$$

Problema Ejemplo

Cuántos arrays de n elementos de números positivos tienen máximo = 10?

Versión 2D

3				
2				
1				
0				
	0	1	2	3

3	8	15	23	31
2	7	12	19	24
1	6	8	13	17
0	3	4	6	7
	0	1	2	3

Versión 2D

3	●	●●	●	●●●
2	●	●●●	●●	●
1	●●●	●	●●●	●●●
0	●●●	●	●●	●
	0	1	2	3

3	8	15	23	31
2	7	12	19	24
1	6	8	13	17
0	3	4	6	7
	0	1	2	3

3				
2		-1	+1	
1		+1	-1	
0				
	0	1	2	3

$$A_{i,j} = S_{i,j} - S_{i-1,j} - S_{i,j-1} + S_{i-1,j-1}$$

Problema Ejemplo

Cuántos arrays de n elementos de números positivos tienen máximo = 10 y mínimo = 3?

$$A_I = \sum_{v \in C} S_{I-v} \cdot (-1)^{\text{cant de 1s en } v}$$

Con $I = (i_1, i_2, \dots, i_n)$, y C son los 2^n vectores de 0 y 1.

Fin!



reedef.dev/feedback