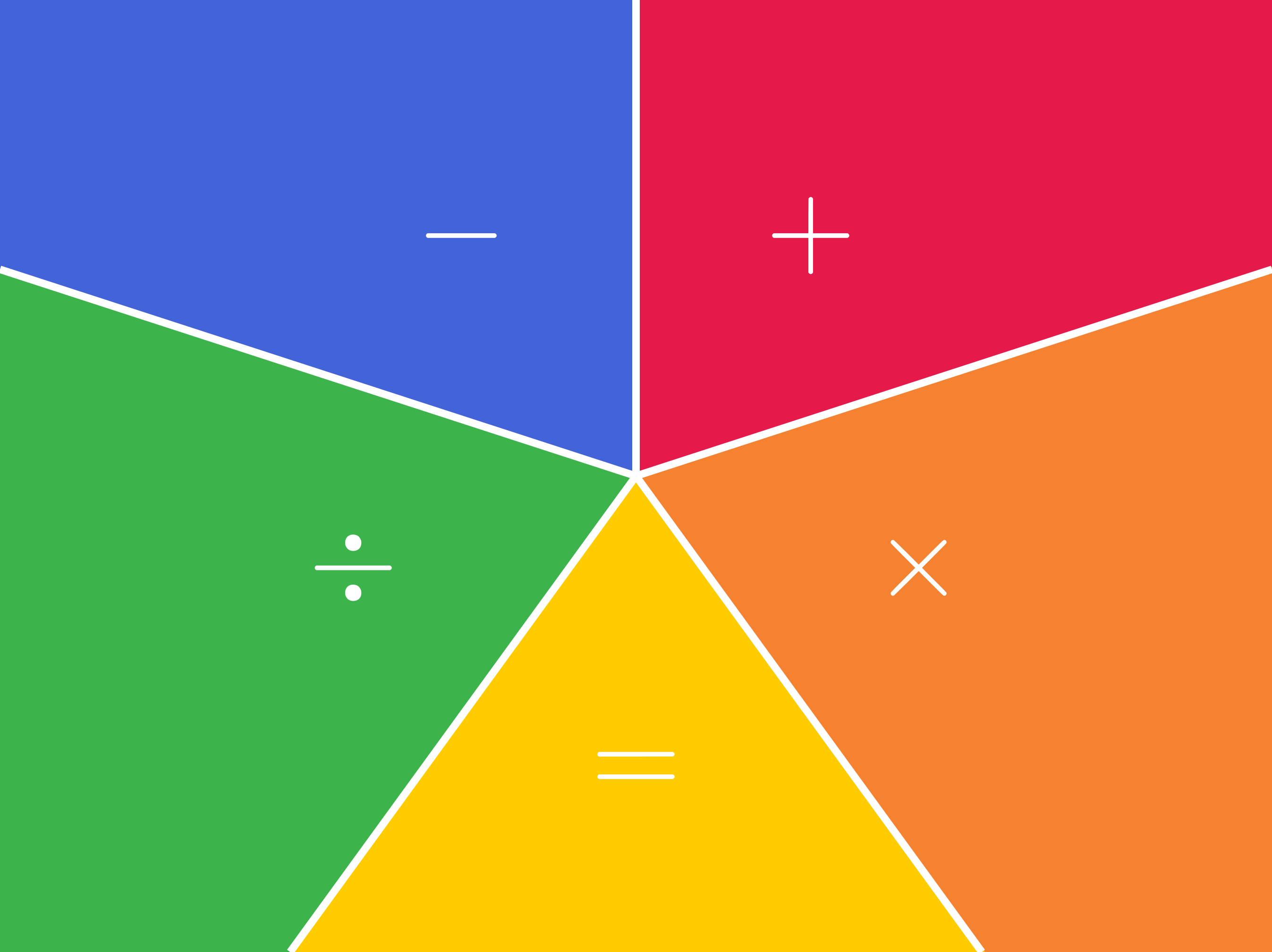
Combinatoria

Charles

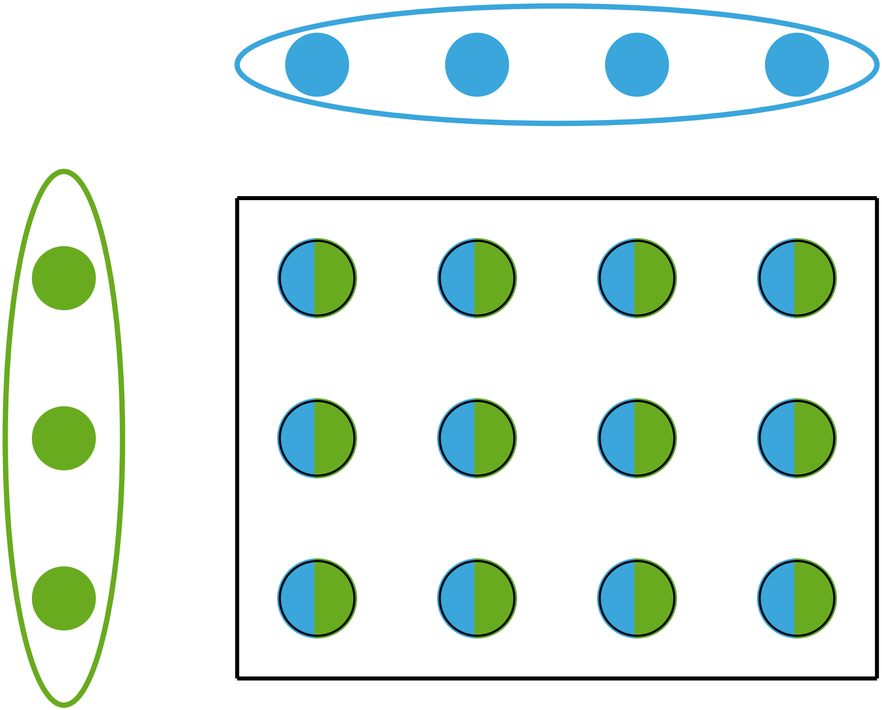
Training Camp

Medellín 2026

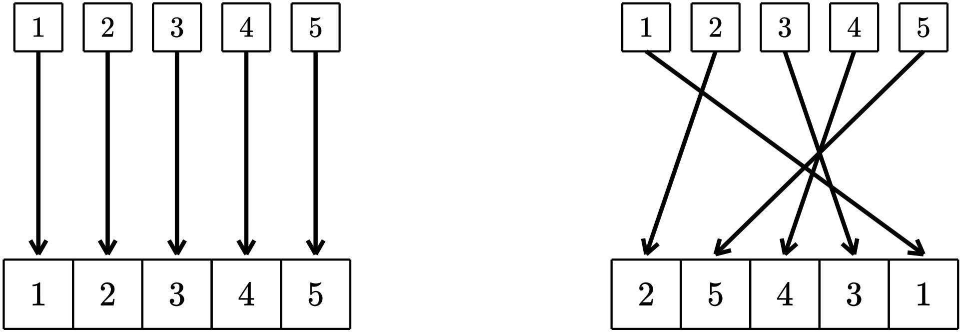
Qué es? Contar cosas.



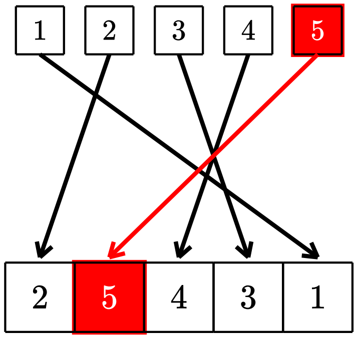
Producto Cartesiano

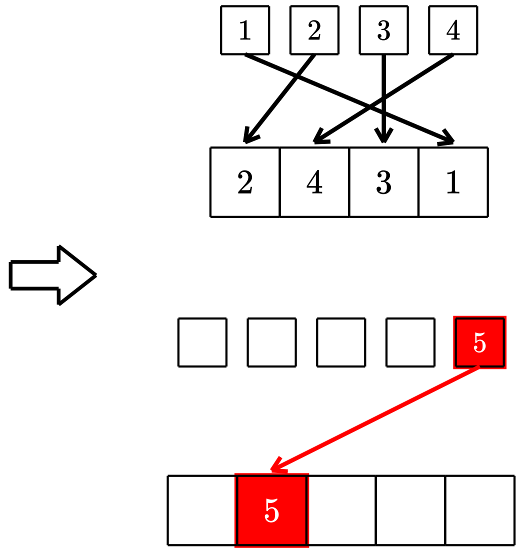


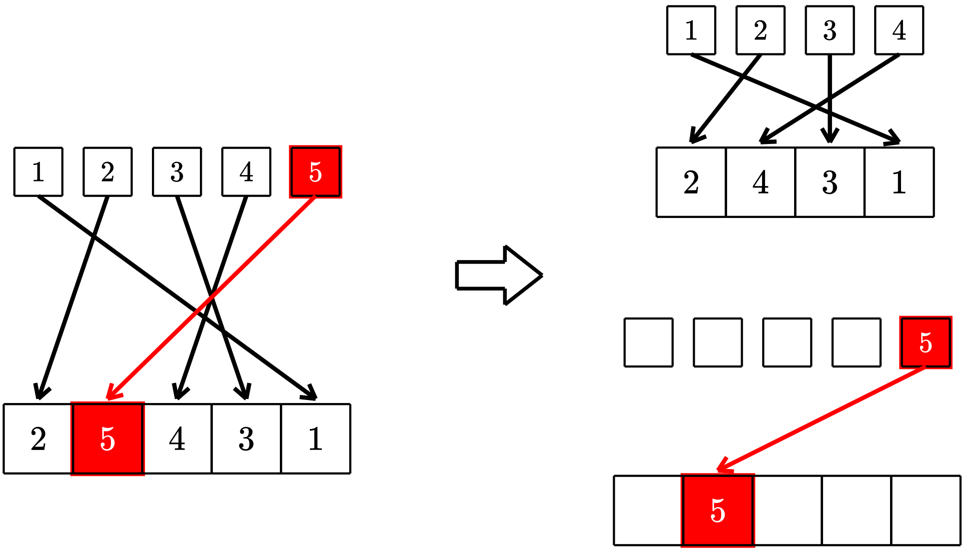
Permutaciones



Conteo Inductivo

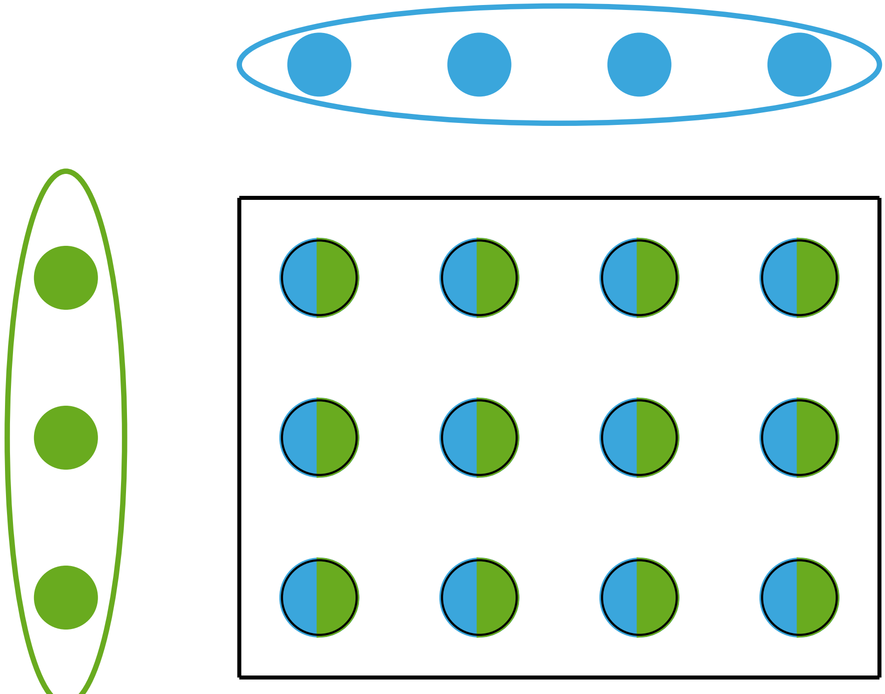




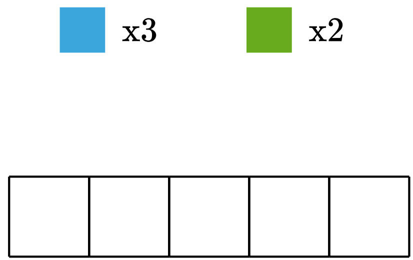


$$\text{perm}(5) = 5 \cdot \text{perm}(4)$$

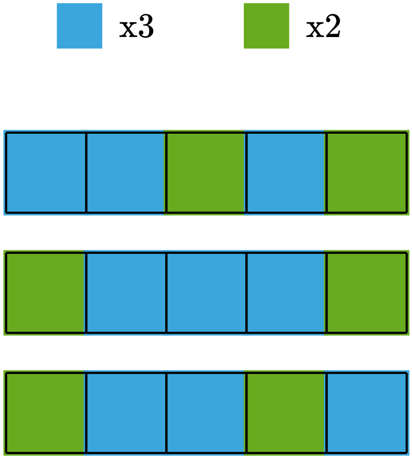
Doble Conteo



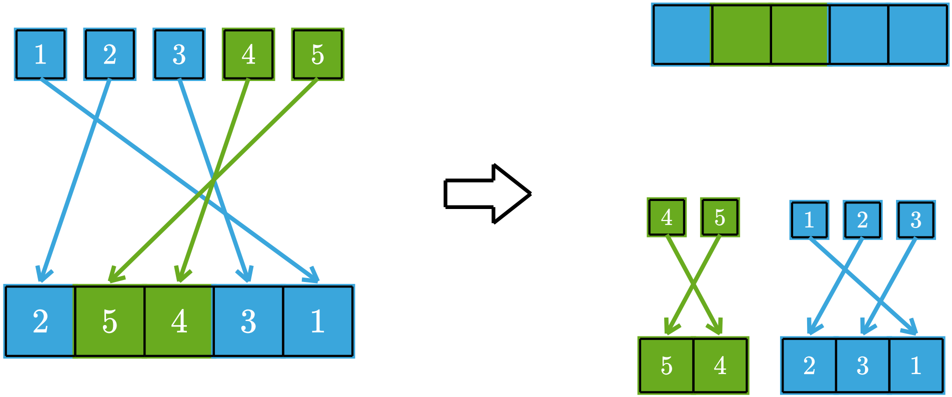
Coeficientes binomiales



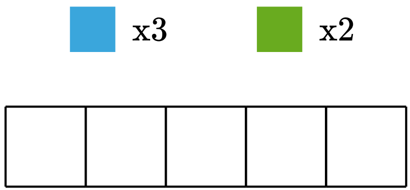
Coeficientes binomiales



Double Counting



Double Counting

$\#\Big($  $\Big) \cdot 3! \cdot 2! = 5!$

$$\binom{5}{3} = \frac{5!}{3! \cdot 2!}$$

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

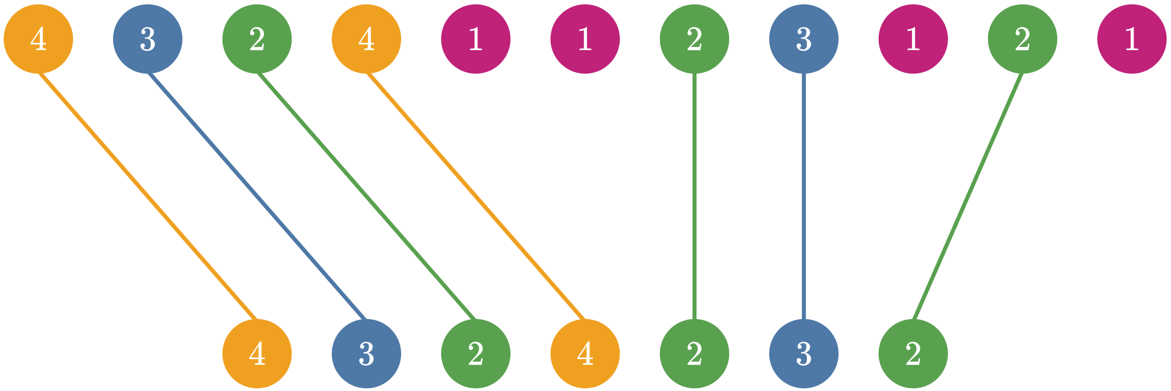
Kyoya and Colored Balls

https://codeforces.com/problemset/problem/553/A

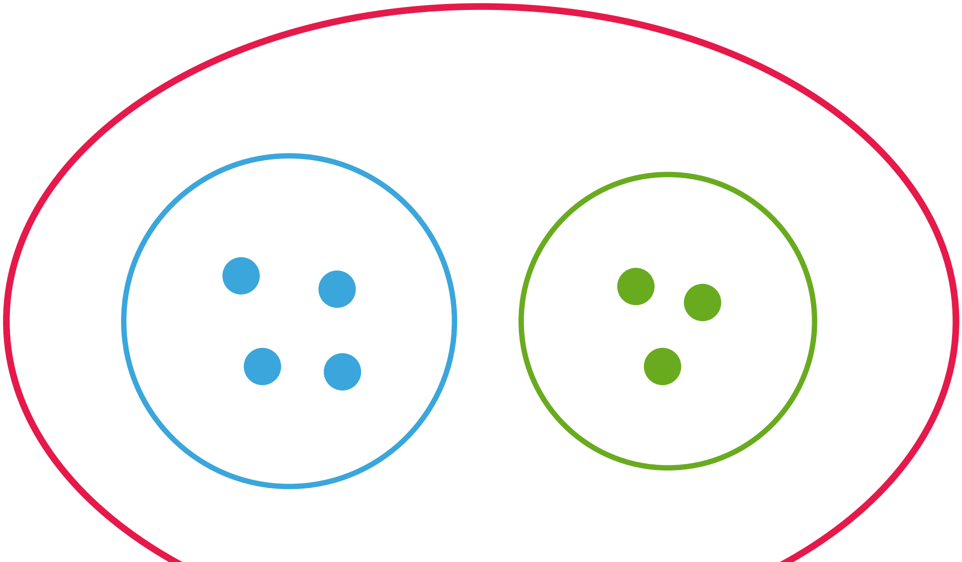
Se tienen k colores y c_i (con $1 \leq i \leq k$) bolitas de cada color. Se las quiere poner en fila tal que la última bolita del color i esté antes que la última bolita del color $i - 1$.

Kyoya and Colored Balls

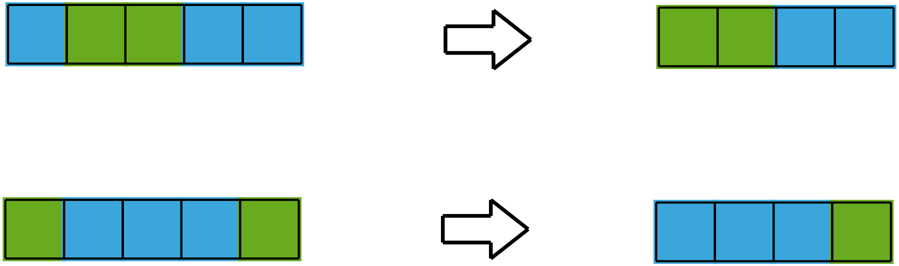
Si sacamos los 1, queda un array válido con un color menos.



Separación en Casos



Triángulo de Pascal



$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Triángulo de Pascal

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

$$\binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$$

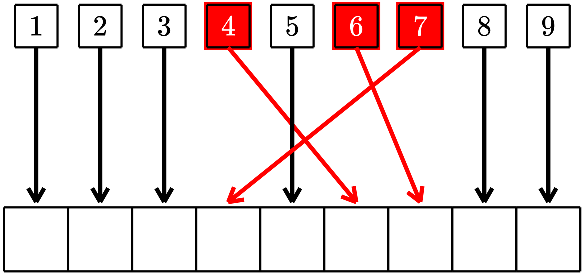
Triángulo de Pascal

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

Problema de Ejemplo

Una permutación es casi identidad si todos excepto a lo sumo k de los números van a su propia caja.

$$n \leq 10^4, k \leq 4$$

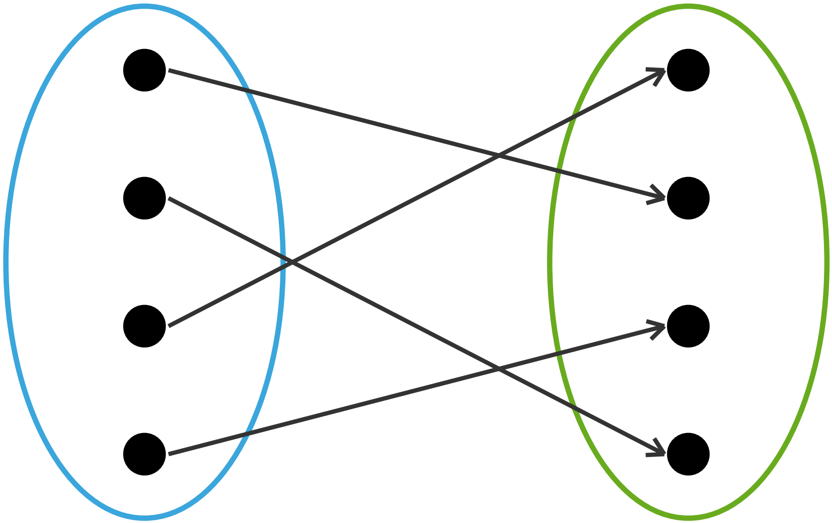


$$\binom{n}{0} \cdot 0! + \binom{n}{1} \cdot 1! + \binom{n}{2} \cdot 2! + \binom{n}{3} \cdot 3! + \binom{n}{4} \cdot 4!$$

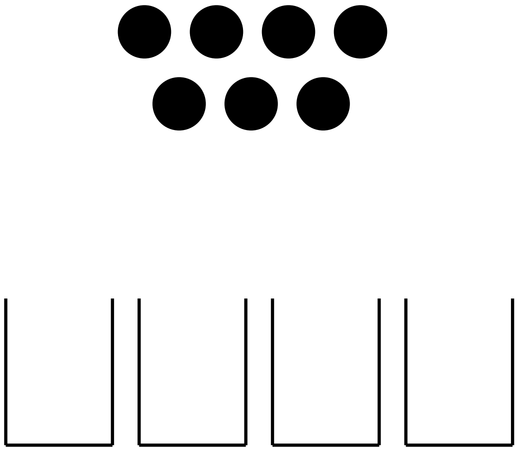
No Anda!

$$\binom{n}{0} \cdot D_0 + \binom{n}{1} \cdot D_1 + \binom{n}{2} \cdot D_2 + \binom{n}{3} \cdot D_3 + \binom{n}{4} \cdot D_4$$

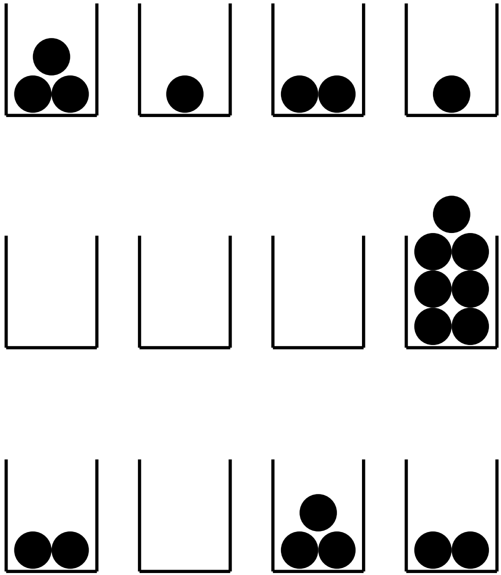
Isomorfismo



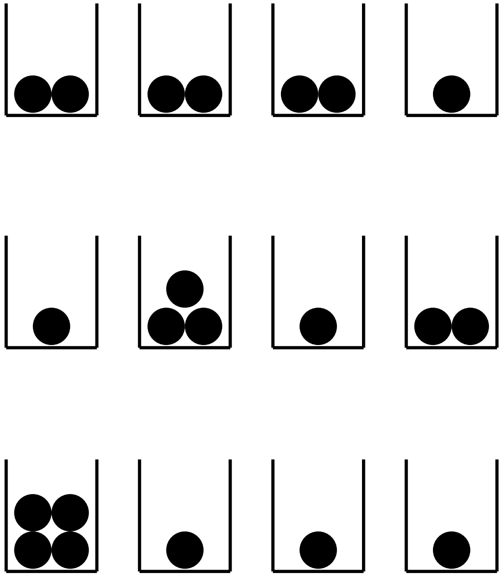
Bolitas y Cajitas



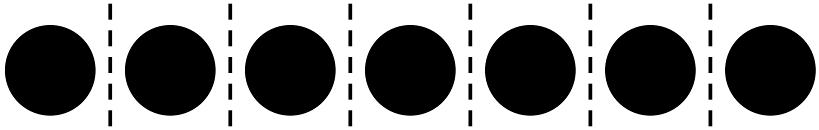
Bolitas y Cajitas



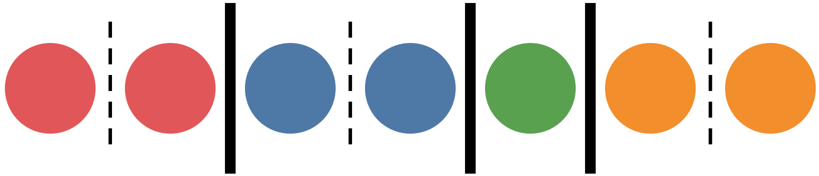
Bolitas y Cajitas (no vacías)



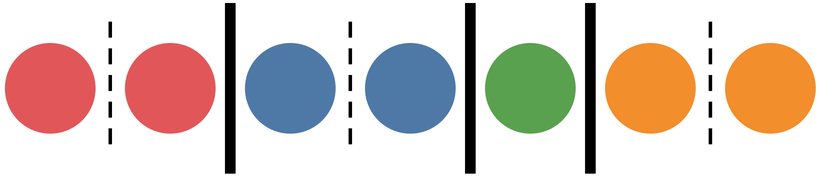
Bolitas y Palitos



Bolitas y Palitos



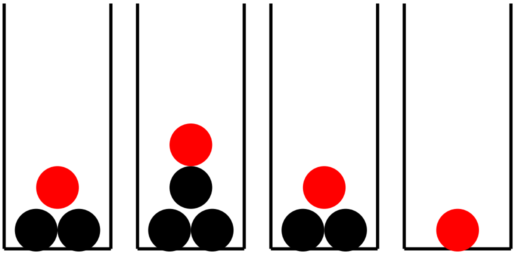
Bolitas y Palitos



$$\text{ByCNV}(7,4) = \binom{6}{3}$$

$$\text{ByCNV}(b,c) = \binom{b-1}{c-1}$$

Bolitas y Cajitas (con vacías)

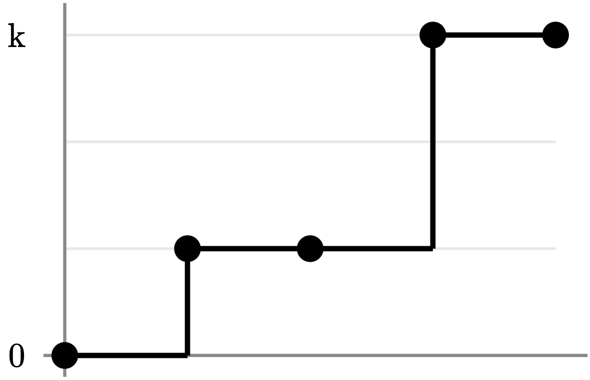


$$\mathrm{ByC}(7,4) = \mathrm{ByCNV}(11,4) = \binom{10}{3}$$

$$\mathrm{ByC}(b,c) = \mathrm{ByCNV}(b+c,c) = \binom{b+c-1}{c-1}$$

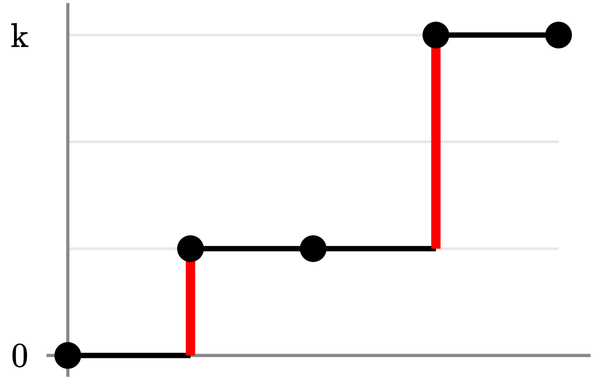
Secuencias no decrecientes

Contar las secuencias no decrecientes de largo n que empiezan en 0 y terminan en k.



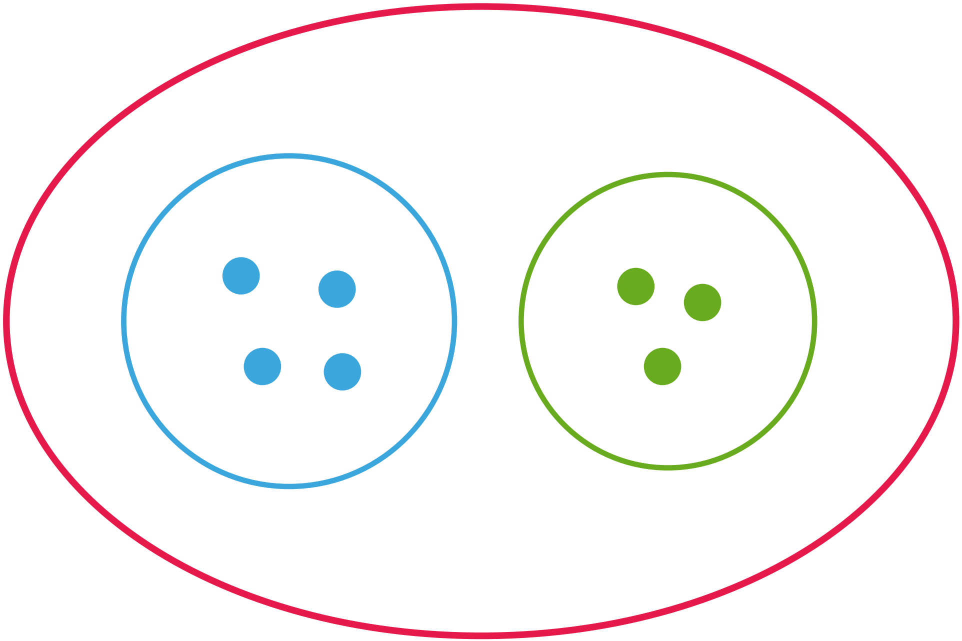
Secuencias no decrecientes

La secuencia queda determinada por sus $n-1$ saltos $d_i = a_{i+1} - a_i$:
son $n-1$ enteros $d_i \geq 0$ que suman k.



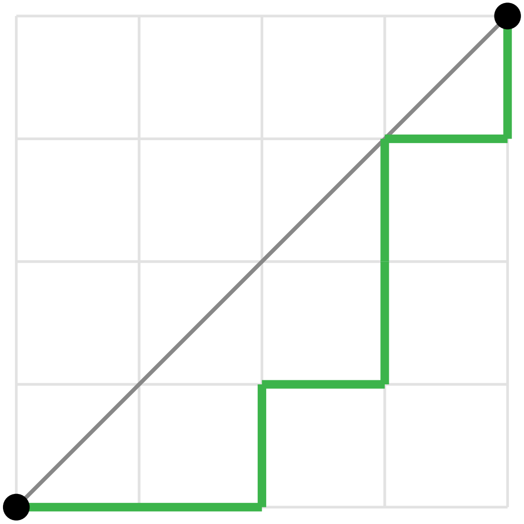
$$\# = \mathrm{ByC}(k, n-1) = \binom{n+k-2}{k}$$

Contar el Complemento



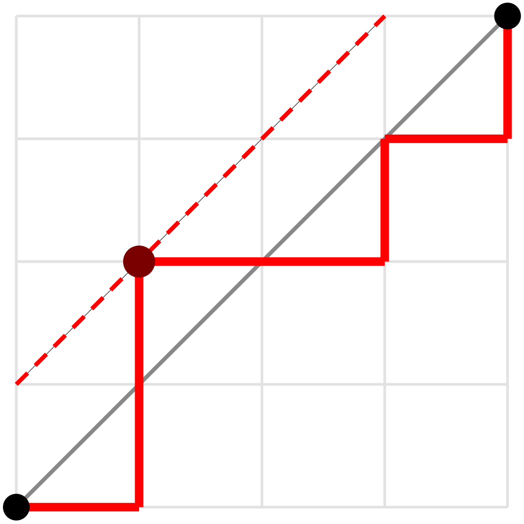
Contar el Complemento: Caminos de Dyck

Un camino monótono de (0,0) a (n,n) da pasos a la derecha y arriba.
Cuántos quedan siempre (débilmente) por debajo de la diagonal?

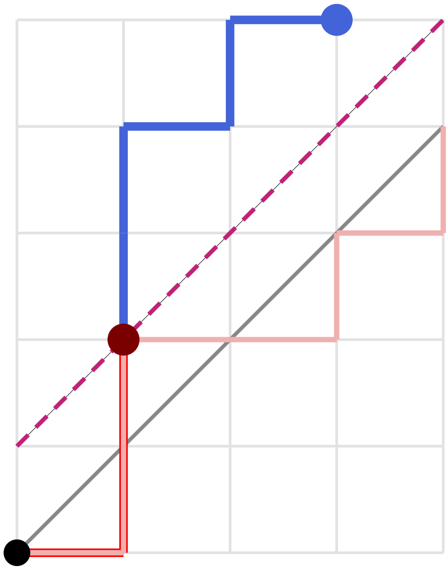


Contar el Complemento: Caminos de Dyck

En total hay $\binom{2n}{n}$ caminos monótonos.
Contamos los malos: los que cruzan, o sea tocan la recta $y = x + 1$.



Contar el Complemento: Caminos de Dyck



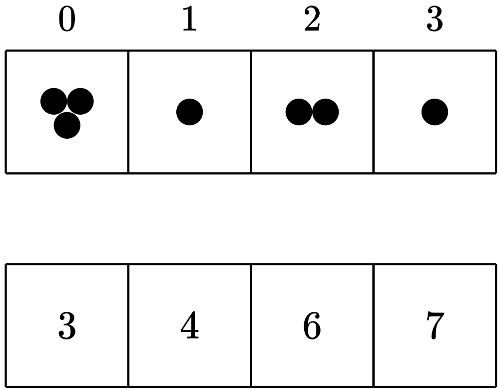
$$\#\ \text{buenos} = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1}\binom{2n}{n}$$

Break

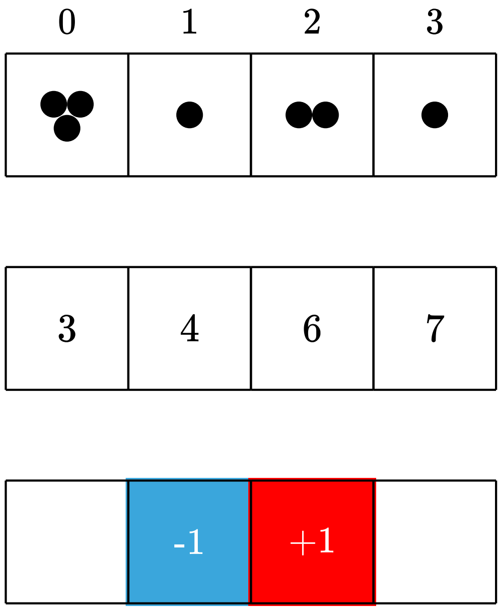
10 min

Inclusion-Exclusion

Inclusión Exclusión



Inclusión Exclusión

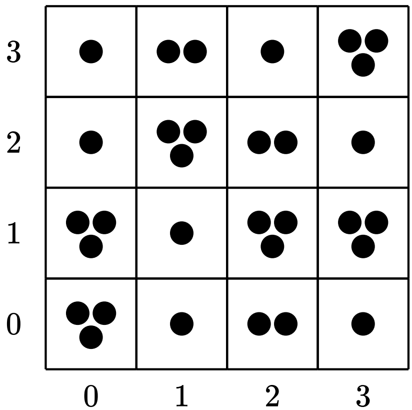


$$A[i] = S[i] - S[i-1]$$

Problema Ejemplo

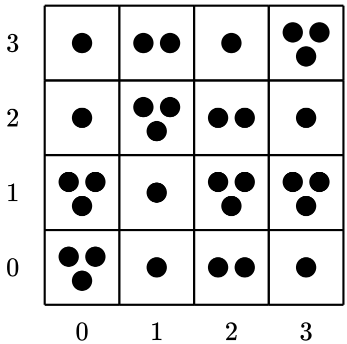
Cuántos arrays de n elementos de números positivos tienen máximo $= 10$?

Versión 2D

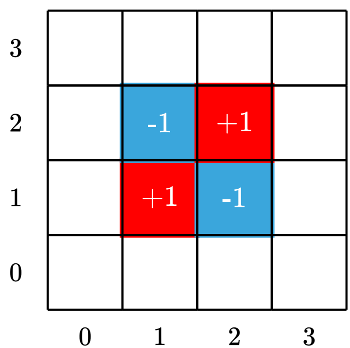


3	8	15	23	31
2	7	12	19	24
1	6	8	13	17
0	3	4	6	7
	0	1	2	3

Versión 2D



3	8	15	23	31
2	7	12	19	24
1	6	8	13	17
0	3	4	6	7
	0	1	2	3



$$A_{i,j} = S_{i,j} - S_{i-1,j} - S_{i,j-1} + S_{i-1,j-1}$$

Problema Ejemplo

Cuántos arrays de n elementos de números positivos tienen máximo $= 10$ y mínimo $= 3$?

Versión ND

$$A_I = \sum_{v \in C} S_{I - v} \cdot (-1)^{\text{cant de 1s en v}}$$

Con $I = (i_1, i_2, ..., i_n)$, y C son los 2^n vectores de 0 y 1.

Fin!

reedef.dev/feedback