Combinatoria

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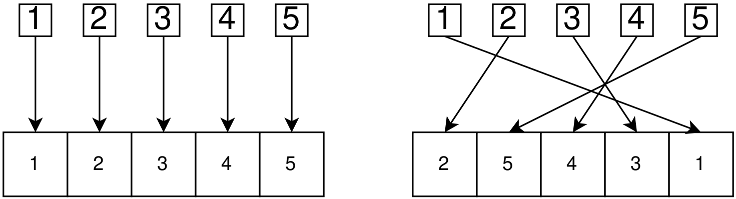
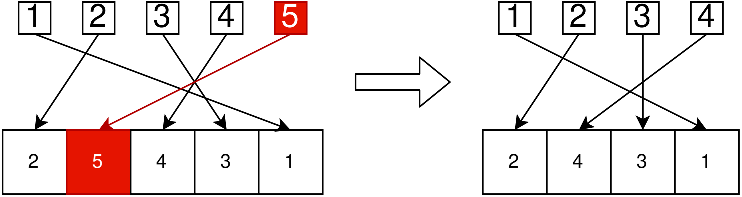
Combinatoria

Qué es? Contar cosas.

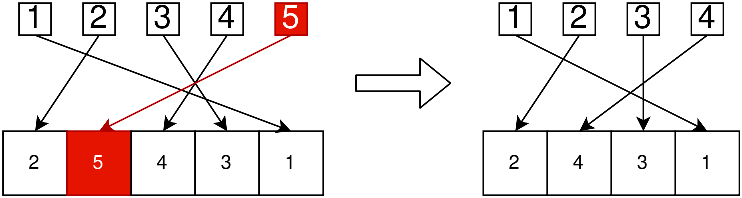
Permutaciones



Conteo Inductivo

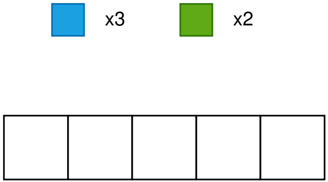


Conteo Inductivo

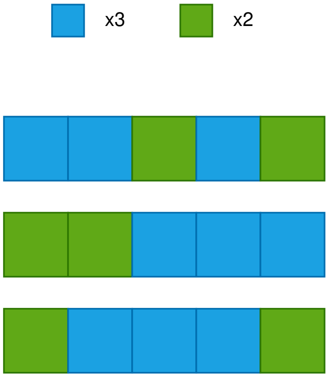


$$5! = 5 \cdot 4!$$

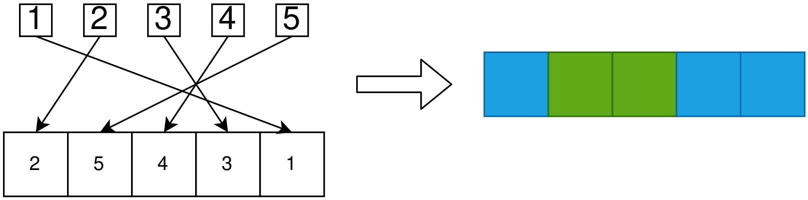
Double Counting: Coeficientes binomiales



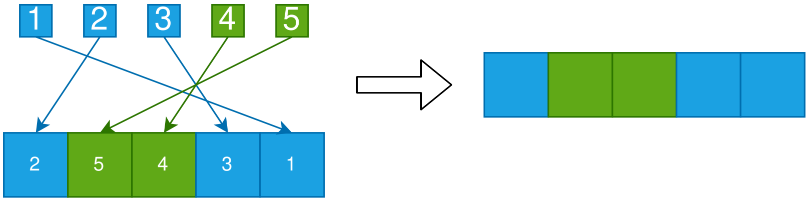
Double Counting: Coeficientes binomiales



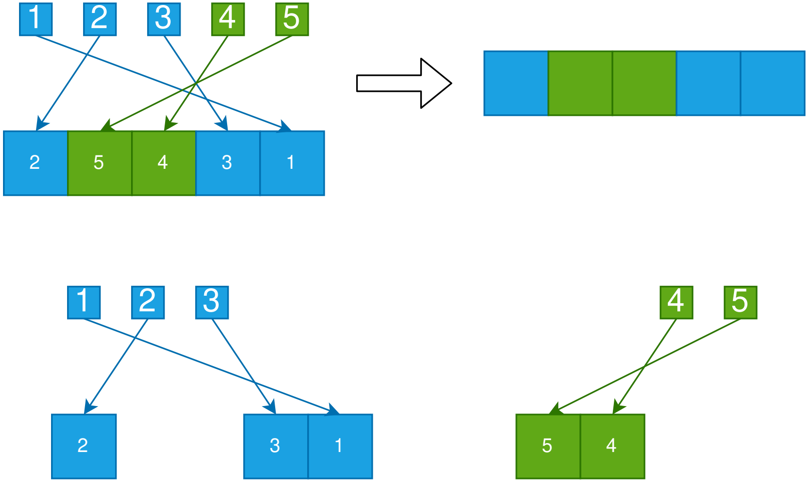
Double Counting: Coeficientes binomiales

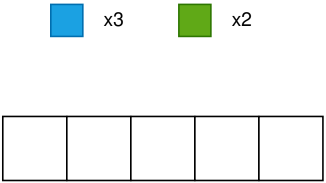


Double Counting: Coeficientes binomiales



Double Counting: Coeficientes binomiales



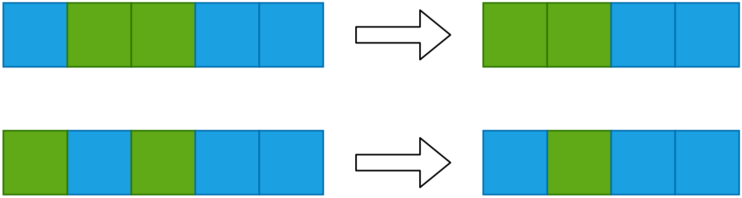
$\# \Bigg($ 

$\Bigg) \cdot 3! \cdot 2! = 5!$

$$\binom{5}{3} = \frac{5!}{3! \cdot 2!}$$

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

Triángulo de Pascal



$$\binom{5}{3} = \binom{4}{2} + \binom{4}{3}$$

Triángulo de Pascal

$$
\begin{array}{ccccccccccc}
 & & & & & \binom{0}{0} & & & & & \\
 & & & & \binom{1}{0} & & \binom{1}{1} & & & & \\
 & & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & & \\
 & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & & \\
 & \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} & \\
\binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5}
\end{array}
$$

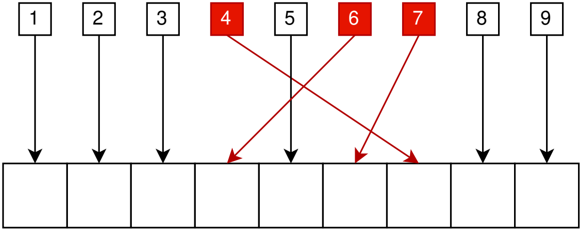
Triángulo de Pascal

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

Problema de Ejemplo

Una permutación es *casi identidad* si todos excepto a lo sumo k de los números van a su propia caja.

$$n \leq 10^4, k \leq 4$$



$$\binom{n}{0} \cdot 0! + \binom{n}{1} \cdot 1! + \binom{n}{2} \cdot 2! + \binom{n}{3} \cdot 3! + \binom{n}{4} \cdot 4!$$

No Anda!

$$\binom{n}{0} \cdot D_0 + \binom{n}{1} \cdot D_1 + \binom{n}{2} \cdot D_2 + \binom{n}{3} \cdot D_3 + \binom{n}{4} \cdot D_4$$

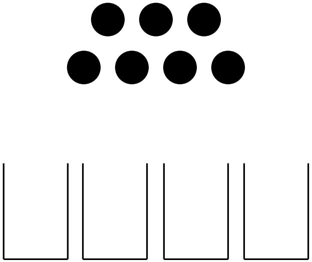
Computar Derangements

Computar Choose

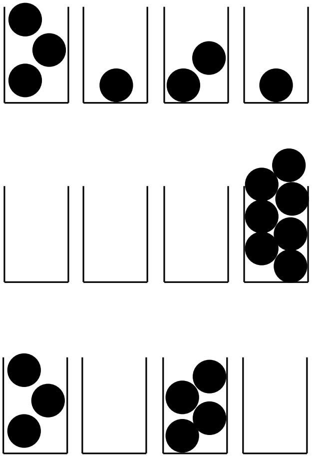
$$\binom{n}{4} = \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-4) \cdot (n-5) \cdot \ldots \cdot 2 \cdot 1}{4! \cdot (n-4) \cdot (n-5) \cdot \ldots \cdot 2 \cdot 1}$$

$$= \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3)}{4!}$$

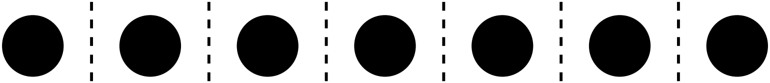
Bolitas y Cajitas



Bolitas y Cajitas



Bolitas y Palitos



Bolitas y Palitos

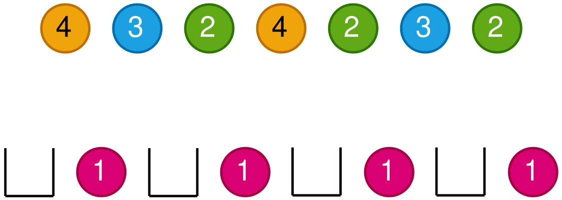
$$\mathrm{ByC}(7,4) = \binom{10}{3}$$

$$\mathrm{ByC}(b,c) = \binom{b+c-1}{c-1}$$

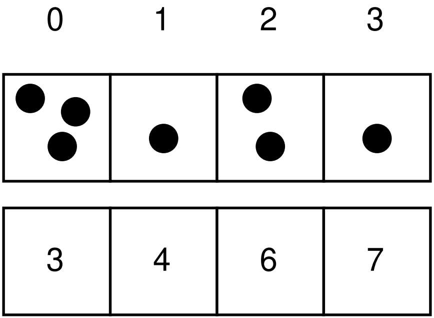
Kyoya and Colored Balls

https://codeforces.com/problemset/problem/553/A
Se tienen k colores y c_i (con $1 \leq i \leq k$) bolitas de cada color. Se las quiere poner en fila tal que la última bolita del color i esté antes que la última bolita del color $i - 1$.

Kyoya and Colored Balls



Inclusión Exclusión



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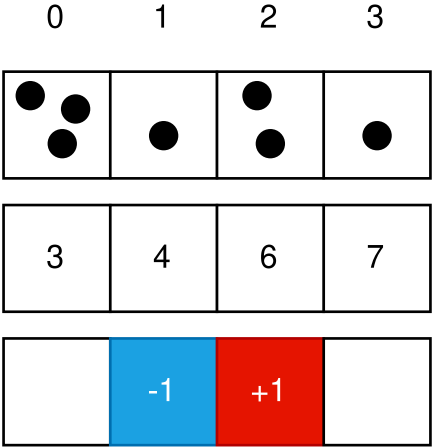
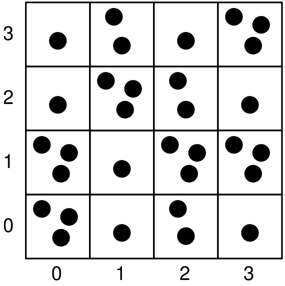


$$A[i] = S[i] - S[i-1]$$

Problema Ejemplo

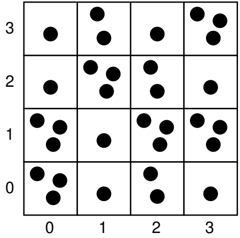
Cuántos arrays de n elementos de números positivos tienen máximo $= 10$?

Versión 2D

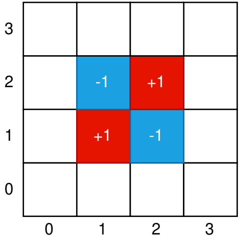


3	8	15	23	31
2	7	12	19	24
1	6	8	13	17
0	3	4	6	7
	0	1	2	3

Versión 2D



	0	1	2	3
3	8	15	23	31
2	7	12	19	24
1	6	8	13	17
0	3	4	6	7



$$A_{(i,j)} = S_{(i,j)} - S_{(i-1,j)} - S_{(i,j-1)} + S_{(i-1,j-1)}$$

Versión 3D

$$\begin{aligned}A_{(i,j,k)} =& S_{(i,j,k)} - S_{(i-1,j,k)}\\ &- S_{(i,j-1,k)} + S_{(i-1,j-1,k)}\\ &- S_{(i,j,k-1)} + S_{(i-1,j,k-1)}\\ &+ S_{(i,j-1,k-1)} - S_{(i-1,j-1,k-1)}\end{aligned}$$

$$A_{(i,j,k)} = \sum_{v \in C} S_{(i,j,k)-v} \cdot (-1)^{\text{cant de 1s en } v}$$

$$\begin{aligned}C = \{&(0,0,0),(0,0,1),(0,1,0),(0,1,1),\\ &(1,0,0),(1,0,1),(1,1,0),(1,1,1)\}\end{aligned}$$

Versión ND

$$A_I = \sum_{v \in C} S_{I-v} \cdot (-1)^{\text{cant de 1s en } v}$$

Con $I = (i_1, i_2, \cdots, i_n)$, y C son los 2^n vectores de 0 y 1.